AN APPLICATION OF GENETIC ALGORITHM TO THE INDUCTIVE ANGULAR DISPLACEMENT TRANSDUCER OPTIMIZATION

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Abstract: The aim of this work is by optimizing the shape and dimensions of the armature of Differential Inductive Transducer (DIT) to widen the linear section of its transfer function. The inverse problem that corresponds to the optimization was solved using the Genetic Algorithm (GA). The Finite Element Method (FEM) was applied to determine the sensor’s inductance at every single value of the armature angular displacement. The obtained results illustrate the efficiency of the mutual work of GA and FEM in the electromagnetic device design.

Keywords: Inverse problem, Shape optimization, Genetic algorithm, Finite element method.

INTRODUCTION

The mechanical spectroscopy is a method, for identifying the mechanical losses and structural changes in metals and alloys during their processing and loading. The installations, used for these investigations contain a torsion pendulum, and a torque generator. During the experiment, the parameters of the pendulum oscillations are measured by means of angular displacement transducer and digitizer. At the next stage, these data is processed to identify the mechanical losses, respectively the internal friction of the tested materials. The correct identification of the mechanical losses strongly depends on the precise measurement of the pendulum deviation.

Depending on the applied torques and the properties of the tested materials, the installations cause angular displacements in the range of ±10 degrees. This allows to accept that the Desired Working Range (DWR) of their angular transducer to be at least DWR=20 degrees. In addition to the requirements for accuracy and the width of the working range, the peculiarities of the installation’s working conditions impose the following demands to the used transducer:

- Continuous output signal;
- Lack of mechanical contact to the pendulum;
- Working in high temperature media;
- Insensitiveness to the surrounding electromagnetic disturbances;
- Insensitiveness to the contamination;
- Linearity of the output characteristic;
- Independent to the speed of swinging.

Among the known angular sensors the Inductive Transducers (IT) totally fulfill the above requirements adding considerable reliability and long live. Unfortunately these transducers have sensitivity that depends on the magnitude of the angular displacement. The linear section of their transfer function corresponds to relatively narrow working range of several degrees in displacement.

This work was inspired from the idea to optimize the shape and dimensions of the armature of an IT, so as to obtain a sensor with good linearity and wide working range. The described optimization task is an electromagnetic inverse problem. Because in electromagnetics these problems are usually ill-posed, some stochastic optimization method such as: GA, Simulated Annealing, or Evolution Strategy [1-3] has to be used. The applications of these methods rely on the existence of numerical method for the solution of the corresponded forward problem.

In this paper, the GA searching technique and FEM was employed to accomplish the optimization. FEM works as forward problem solver. During the solution, all coil parameters were accepted to be unchangeable. The optimization was conducted in order to find the dimensions of the armature that ensure the widest linear section of the transducer’s transfer function.

INDUCTIVE TRANSDUCER’S PECULIARITIES

A variation of the IT that is used for the torsion pendulum swinging measurements is shown in Fig. 1. Two coils were placed around a ferromagnetic tooted wheel, joined at the end of the swinging pendulum. The positions of coils are chosen in a manner that provides contrary alteration of the impedances. With the wheel movement the reluctance of the magnetic circuit through one of the coils decreases, and the reluctance of the other increases.

![Fig. 1. - The Inductive Transducer](image-url)
Both coils are connected in series to a sinusoidal current source. The low values of the coils’ dimensions and the used excitation frequency define their resistance to be less than 100 time compared to their reactance. In addition to that the low values of the used supplied current, and the presence of the air gap between the wheel and coils, determine low values of the power loses inside the armature. Hence the changes in coils’ impedance, according to the swinging of the pendulum, are totally dependent to the changes of their inductance.

The dependence of the coils’ impedances $Z_A$ and $Z_B$, to the angular turn of the wheel is strongly nonlinear, Fig. 2a. The balance position of the wheel equalizes $Z_A$ and $Z_B$ and is accepted to correspond to the $\alpha = 0$ angular displacement. To improve the linearity of conversion a DIT is introduced [4]. They transformed the angular displacement of the wheel to the differential impedance $DZ$, where $(DZ = Z_A - Z_B)$. This is done by processing the voltage drops at coils’ terminals. The resulting characteristic $DZ(\alpha)$, Fig. 2b, is symmetrical regarding to the displacement $\alpha = 0$.

The comparison of the output characteristic of the IT (Fig. 2a), to the same characteristic of the DIT (Fig. 2b), shows, that DITs are sensors with better linearity and worse sensitivity. The suitable output characteristic of the DITs, combined with continuous output signal with infinite resolution keeps them among the most popular type of angular position sensors.

Nevertheless their advantages, the $DZ(\alpha)$ characteristic of the DIT is nonlinear as well. There is a section in it that differs from the straight line, less than one sufficiently small number $\beta$. The difference between the values of the input influence $\alpha_{ep1}$ and $\alpha_{ep2}$, which correspond to the end points of this section, defines the transducer’s working range of displacement:

$$WR = \alpha_{ep1} - \alpha_{ep2}$$  \hspace{1cm} (1)$$

The width of the $WR$, for most of the commercial DITs is about 50% of the range of the possible $\alpha$ values. It depends on the dimensions and properties of the moving armature [4].

### DIT’S NUMERICAL MODELING AND ANALYSIS

In this work the interaction between the coils and the rotating DIT’s armature is analyzed numerically using FEM. The correspondent to the problem PDE was solved under following general simplifications:

- The field is considered two dimensional, plane-parallel;
- All used materials are homogeneous. The hysteresis properties of ferromagnetic materials are neglected;
- Electromagnetic field attenuates entirely inside the boundaries ABCD;

The constructed numerical model, Fig. 3, contains a steel toothed wheel that rotates in front of the coils $A$ and $B$. The air gap with width $g$, separates the coils and the wheel. Both coils were supplied by a sinusoidal current source. They have square cross section, equal number of turns and identical ferromagnetic cores. The wheel is defined with its inner and outer radius, $R_1$ and $R_2$, and with the angular dimensions of its teeth, $\alpha_1$ and $\alpha_2$. The software package FEMM [5] is used to collect information for field distribution at $n$ different position of the wheel, uniformly distributed around its balance position. The obtained results for magnetic vector potential are used to calculate the impedances $Z_A$ and $Z_B$.

The parameterized in this manner model allow modifying the DIT’s construction by changing the values of the parameters: $g$, $R_1$, $R_2$, $\alpha_1$ and $\alpha_2$. To automate the model construction and impedances calculations at every positions of the wheel, a special shell program was written using the scripting language LUA. The availability of such shell is very important for the solution of the inverse problem. It saves a lot of work on preparation of the FEMM models, drives the simulation, and reduces to zero the operator’s errors. The automatically generated mesh has typically 10000 triangular elements of first order, which ensure the necessary precision.
OPTIMIZATION ALGORITHM

The application of GA, as an electromagnetic inverse problem solver, requires the boundaries of the model variable spaces and an Objective Function (OF) to be defined in advance. The solution of the inverse problem, are these values of the variables that provide an extreme of the OF. Usually this function represents the difference between the desired and the current value of some parameter of the examined device.

In electromagnetics, the constructed OF are multiextreme as a rule. Their global extreme leads to the exact solution of the considered inverse problem. Among the known stochastic methods GA is preferable mainly due to its ability to avoid trapping in the local extreme of OF. It is based on the best individual survival and explores the solution space of the OF, keeping the best ones.

![Fig. 2. - The numerical model](image)

The optimization goal in this work is to obtain these values of the transducer’s armature dimensions that guarantee maximum width of the WR. To accomplish this, GA is employed to search the maximum of an OF defined by: \( OF = WR \). The transducer’s WR is computed in the following succession:

- The forward problem is solved numerically for \( n \) different displacements, uniformly distributed in the whole range of possible positions of the moving wheel. The transducer’s \( DZ \), for every single displacement \( \alpha_g \) is computed from the calculated coils’ impedances \( ZA(\alpha_g) \) and \( ZB(\alpha_g) \):

  \[
  DZ(\alpha_g) = ZA(\alpha_g) - ZB(\alpha_g); \quad (2)
  \]

- The position \( \alpha_r \) that corresponds to the \( DZ = 0 \) is than recognized;

- At the vicinity of the \( \alpha_r \) position, two different contiguous displacements \( \alpha_r \) and \( \alpha_K \) were selected and the sensitivity \( Sn \) was estimated:

  \[
  Sn = \frac{DZ(\alpha_r) - DZ(\alpha_K)}{\alpha_r - \alpha_K}; \quad (3)
  \]

- With the sensitivity \( Sn \), the transducer’s nonlinearity is calculated for every single displacement \( \alpha_g \) with:

  \[
  NL(\alpha_g) = \frac{DZ(\alpha_g) - Sn \cdot \alpha_g}{\alpha_g}; \quad (4)
  \]

- The obtained results were compared to the maximum possible value of the transfer function nonlinearity \( \beta \). The comparison results were converted to the angular displacement, and stored in the \{ANL\} array, using the notation:

  \[
  anl_g = \begin{cases} 
  \alpha_g, & NL(\alpha_g) \leq \beta \\
  0, & NL(\alpha_g) > \beta
  \end{cases}; \quad (5)
  \]
Transducer’s $WR$, respectively the inverse problem $OF’$ is calculated by:

$$OF = WR = \max \{ANL\} - \min \{ANL\}.$$  \hspace{1cm} (6)

GA searches the global maximum of the $OF$ by changing the values of the five parameters inside the preliminary defined limits. The representation of these parameters was done using decimal coding, and the obtained genetic individual is a numerical value for the $g$, $R1$, $R2$, $\alpha_1$, $\alpha_2$. The used genetic operators are: selection, cross-over and mutation. At the selection phase the solutions that provides best values for the $OF$ were founded and preserved. New individuals were introduced with the help of cross-over and mutation operators. The $OF$ for each individual was determined after $n$ runs of forward problem solution. In spite of the fact that large population is better for the genetic process development, the large amount of computer time used for every individual imposes to limit the population size. The stop condition also exists, and ends the iterative process when reaches the generation number established in the beginning.

**Preliminaries and Optimization Results**

The considered in this work DIT for mechanical spectroscopy was optimized using GA and FEM. Considering the pointed in the beginning values for $DWR$ of the optimized DIT and the peculiarity of its transfer function, the numerical simulation have to be conducted at wider Range of Possible Angular Displacements ($RPAD$) of the pendulum.

The width of $RPAD$ was accepted to be 30 degrees, or $\pm 15 \text{deg}$ in respect to the wheel’s balance position. The changes of the DIT’s $DZ$ were computed at $n=31$ positions of the pendulum, uniformly distributed in the $RPAD$.

Both coils have $4 \text{ mm}^2$ cross-section areas, and ferromagnetic cores, $2 \times 20 \text{ mm}$ in dimensions, made of N26 material. They were connected in series and supplied by $10 \text{mA}$, $20 \text{kHz}$ sinusoidal current source. During optimization the excitation conditions were accepted to be unchangeable.

The applied GA [6], works in floating-point representation of variables, and is organized to search the maximum of the defined $OF$. The initial population is randomly created and contains 50 individuals. Following the principles developed by Michalewicz [7], the selection, crossover and mutation operators are used to create new individuals. In this work GA works with 20 individuals in generation and ended after 50 generations. The algorithm’s search direction is specified by the values of the $OF$, which is computed with results from the forward problem solutions and the chosen nonlinearity $\beta = 0.01$. During the solution of the considered inverse problem, the selected five parameters could vary inside the boundaries, pointed in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$g$</th>
<th>$R1$</th>
<th>$R2$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>unit</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>deg</td>
<td>deg</td>
</tr>
<tr>
<td>Min value</td>
<td>0.7</td>
<td>14</td>
<td>12</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Max value</td>
<td>1.5</td>
<td>18</td>
<td>17</td>
<td>35</td>
<td>35</td>
</tr>
</tbody>
</table>
Fig. 4. - Ferrite cored DIT - transfer function and linearity characteristics
The described optimization technique was applied in order to find these values of the chosen five parameters, which provide the widest WR of the considered DIT. Unfortunately, the obtained results show that DIT with ferrite cored coils has WR significantly lower than the defined DWR.

The optimization was repeated several times irrespective of the duration of the computations. The solution that gives best WR, together with the values of the DIT’s parameters and the average sensitivity Sn* are depicted in Table II. The transfer function and linearity characteristic of the same sensor are shown at Fig. 4.

Table II.
Ferrite cored DIT – results from optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>WR</th>
<th>Sn*</th>
<th>g</th>
<th>R1</th>
<th>R2</th>
<th>α1</th>
<th>α2</th>
</tr>
</thead>
<tbody>
<tr>
<td>unit</td>
<td>deg</td>
<td>Ω/deg</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>deg</td>
<td>deg</td>
</tr>
<tr>
<td>Value</td>
<td>13.1</td>
<td>12.8</td>
<td>1.14</td>
<td>16.5</td>
<td>13.1</td>
<td>29.4</td>
<td>24.2</td>
</tr>
</tbody>
</table>

In order to obtain a sensor that satisfy the requirement for the width of the working range (WR≥DWR), various changes in the coils dimensions were examined. The appropriate numerical models were constructed and the optimization process was carried out. Among the all numerically tested transducers, the sensor without ferrite cores in the coils gives best results according to the WR parameter. The optimization results for this DIT are pointed out in Table III, and its transfer function and linearity characteristic are shown in Fig. 5.

Table III.
Air cored DIT – results from optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>WR</th>
<th>Sn*</th>
<th>g</th>
<th>R1</th>
<th>R2</th>
<th>α1</th>
<th>α2</th>
</tr>
</thead>
<tbody>
<tr>
<td>unit</td>
<td>deg</td>
<td>Ω/deg</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>deg</td>
<td>deg</td>
</tr>
<tr>
<td>Value</td>
<td>20.8</td>
<td>3.8</td>
<td>0.97</td>
<td>15.8</td>
<td>13.7</td>
<td>30.6</td>
<td>25.1</td>
</tr>
</tbody>
</table>

CONCLUSIONS
This work represents the results of inverse problem solution in the field of electromagnetic device optimization. The appropriate numerical model was prepared and parameterized in order to allow all kinds of future changes in geometry. The genetic process controls the model construction and runs the forward problem solver. The information interchange, during the organized mutual work of the GA and FEM, was facilitated by the written for the aim of this work shell program.

Several type of DIT were optimized in order to obtain the sensor with WR>DWR. The investigations started with optimization of a DIT, described at Fig. 3. In spite of the several optimization runs, the obtained results slightly differ from these, pointed in Table 2. Obviously, the good sensitivity of these DITs is combined with narrow working range.

To widen the WR, different changes in coils’ dimensions and positions were made and their optimizations were carried out. Unfortunately the changes in the WR width were insignificant. The situation considerably changes for transducers without ferrite cores in the coils. These transducers offers best results for the WR, Table 3, but their sensitivity is significantly lower. In spite of all, one can say, that the properties and the characteristics of this type of DITs, Fig. 5, totally fulfill the requirements of the mechanical spectroscopy installations.

The chosen in this work technique for electromagnetic device optimization is based on an application of numerical methods for ill-posed inverse problem solution. This approach needs a lot of time and computer resources even in case of 2D solution. Trying to increase the algorithm convergence speed it was noted that it doesn’t depend on number of model variables or the range of their deviation. This allows concluding, that the used approach is suitable for problems with large number of optimization parameters.

REFERENCES

Ivaylo Dolapchiev graduated from the Technical University of Sofia in 1982 with MSc degree in electrical engineering. From 1982 to 1986 he worked as a project engineer at the State Railway Research Institute. In 1986 he became an assistant professor in the Technical University of Sofia, in the Department of Electrical Engineering.

His principal interest is in quantitative NDE and in development of models, algorithms, and sensors. During the past few years he works on applying numerical methods for inverse problems solution in the field of electromagnetic NDT, as well as in optimization of the inductive sensors.